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ABSTRACTS

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LIST OF TALKS

Natalia Adamska
Adam Mickiewicz University

Lower bounds for online Ramsey numbers for paths

Given two graphs G and H, a size Ramsey game is played on the edge set of $K_{\mathbb{N}}$. In every round, Builder selects an edge and Painter colours it red or blue. Builder's goal is to force Painter to create a red copy of G or a blue copy of H as soon as possible. The online (size) Ramsey number $\tilde{r}(G, H)$ is the number of rounds in the game provided Builder and Painter play optimally.

Let P_n denote a path with n vertices. Cyman, Dzido, Lapinskas and Lo [2] proved $\tilde{r}(P_3, P_n) = \lceil 1.25(n-1) \rceil$, $\tilde{r}(P_4, P_n) = 1.4n + O(1)$, $\tilde{r}(P_5, P_n) \geq 1.5(n-1)$ and conjectured, that $\tilde{r}(P_k, P_n) = 1.5n + o(n)$ for any fixed $k \geq 5$. Recently, Mond and Portier [3] disproved that hypothesis by showing that $\tilde{r}(P_{10}, P_n) \geq 1.(6)n - 2$. This matches (up to a constant for a fixed k) the upper bound $\tilde{r}(P_k, P_n) \leq 1.(6)n + 12k$ found by Bednarska-Bzdęga [1]. We improve the result of [3] by showing $\tilde{r}(P_9, P_n) \geq 1.(6)n - 2$. We also show that $\tilde{r}(P_8, P_n) \geq 1.(63)n - 2$ and $\tilde{r}(P_7, P_n) \geq 1.6n - 2$, therefore disproving the conjecture from [2] for $k \geq 7$. Our approach unifies methods used in [2] and [3].

This is joint work with Grzegorz Adamski.

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Sarosh Adenwalla
University of Liverpool

Avoiding monotone arithmetic progressions in permutations of the integers

A monotone arithmetic progression of length k in a sequence of integers, $a_1, a_2, a_3, \ldots$, is a subsequence, $a_{i_1}, a_{i_2}, \ldots, a_{i_k}$, where $i_1 < i_2 < \ldots < i_k$ and $a_{i_j} + d = a_{i_{j+1}}$ for $j = 1, \ldots, k-1$ and a $d \in \mathbb{Z}$. It was shown in [4] that the positive integers could be rearranged to avoid monotone arithmetic progressions of length 5 but not of length 3.

A permutation of the integers avoiding monotone arithmetic progressions of length 6 was constructed in [2]. We use constructions using modular arithmetic to improve on this by constructing a permutation of the integers avoiding monotone arithmetic progressions of length 5. A permutation of the positive integers that avoided monotone arithmetic progressions of length 4 with odd common difference was constructed in [3]. We generalise this result and show that for each $k \geq 1$, there exists a permutation of the positive integers that avoids monotone arithmetic progressions of length 4 with common difference not divisible by 2^k. This is then used to show that there exist subsets of the positive integers with lower density arbitrarily close to 1 that can be rearranged to avoid monotone arithmetic progressions of length 4.

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Pedro Araújo

Czech Technical University in Prague

Semidegree threshold for spanning trees in oriented graphs

We prove that, for any $\gamma > 0$, every n-vertex oriented graph G with $\delta^+(G), \delta^-(G) \geq (3/8 + \gamma)n$ contains every bounded degree tree on n vertices in every possible orientation, as long as n is large enough. This is asymptotically best possible.

In this talk we will focus on the almost spanning case and show how to combine mixing properties of random walks and the regularity lemma in tree embeddings.

This is joint work with Giovanne dos Santos and Maya Stein .

József Balogh

University of Illinois Urbana-Champaign

TURÁN DENSITY OF LONG TIGHT CYCLE MINUS ONE HYPEREDGE

Denote by C_ℓ^- the 3-uniform hypergraph obtained by removing one hyperedge from the tight cycle on ℓ vertices. It is conjectured that the Turán density of C_5^- is $1/4$. In this paper, we make progress toward this conjecture by proving that the Turán density of C_ℓ^- is $1/4$, for every sufficiently large ℓ not divisible by 3. One of the main ingredients of our proof is a forbidden-subhypergraph characterization of the hypergraphs, for which there exists a tournament on the same vertex set such that every hyperedge is a cyclic triangle in this tournament.

A byproduct of our method is a human-checkable proof for the upper bound on the maximum number of almost similar triangles in a planar point set, which was recently proved using the method of flag algebras by Balogh, Clemen, and Lidický.

Joint work with Haoran Luo.

Mindaugas Bloznelis

Vilnius University

Sparse and clustered stationary temporal networks

We present two models of sparse temporal networks that display transitivity - the tendency for vertices sharing a common neighbour to be neighbours of one another. Our first network is a continuous time Markov chain $G = \{G_t = (V, E_t), t \geq 0\}$ whose states are graphs with the common vertex set $V = \{1, \ldots, n\}$. The transitions are defined as follows. Given t, the vertex pairs $\{i, j\} \subset V$ are assigned independent exponential waiting times A_{ij}. At time $t + \min_{ij} A_{ij}$ the pair $\{i_0, j_0\}$ with $A_{i_0 j_0} = \min_{ij} A_{ij}$ toggles its adjacency status. To mimic clustering patterns of sparse real networks we set intensities a_{ij} of exponential times A_{ij} to be negatively correlated with the degrees of the common neighbours of vertices i and j in G_t. Another temporal network is based on a latent Markov chain $H = \{H_t = (V \cup W, E_t), t \geq 0\}$ whose states are bipartite graphs with the bipartition $V \cup W$, where $W = \{1, \ldots, m\}$ is an auxiliary set of attributes/affiliations. Our second network $G' = \{G'_t = (E'_t, V), t \geq 0\}$ is the affiliation network defined by H: vertices $i_1, i_2 \in V$ are adjacent in G'_t whenever i_1 and i_2 have a common neighbour in H_t. We analyze geometric properties of both temporal networks analytically and by numerical simulations and show that networks possess high clustering. They admit tunable degree distribution and clustering coefficients. Our models extend earlier work of [1], [2].

This is joint work with Dominykas Marma

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Simona Boyadzhiyska

University of Birmingham

Simultaneous colourings of graphs

A classic result due to Vizing states that every graph with maximum degree Δ has a proper edge-colouring using at most $\Delta+1$ colours. In this talk, we will discuss a natural generalisation of this problem in which the task is to properly edge-colour several graphs simultaneously. More precisely, given graphs $G_1, \ldots, G_k$ on the same vertex set, we are interested in the minimum number of colours needed to colour the edges of $G_1 \cup \cdots \cup G_k$ so that the colouring induced on each individual graph G_i is proper; we denote this number by $\chi'(G_1, \ldots, G_k)$. This problem was proposed by Cabello and was studied by Bousquet and Durain and subsequently by Cambie. We improve upon their work and show that, for any graphs G_1 and G_2 with maximum degree at most Δ, we have $\chi'(G_1, G_2) = (1 + o(1))\Delta$ as $\Delta \to \infty$. Our ideas generalise to an arbitrary number of graphs, again yielding asymptotically tight results.

This is joint work with Richard Lang, Allan Lo, and Mike Molloy.

Ignacy Buczek

Jagiellonian University

Balanced bipartitions of K_4-free graphs

By balanced bipartite distance of a graph we denote the number of edges that need to be removed to make it balanced bipartite. The main motivation for researching this graph parameter is its connection to the famous sparse half conjecture of Erdős ([1]). In the class of triangle-free graphs, a tight upper bound of $\frac{n^2}{16}$ on the balanced bipartite distance has been proven ([2]). In the class of K_4-free graphs, the problem of proving analogous upper bound is unsolved. We know that it cannot be better than $\frac{n^2}{9}$, as complete balanced tripartite graph attains this exact value. The best known result proves the upper bound of $\frac{n^2}{9}$ for tripartite, K_4-free graphs ([2]). We strictly improve this result by proving the conjecture for graphs with the minimum degree of at least $\frac{n}{2}$. We also derive a couple of arguments which allow relaxing the requirement on the minimum degree to $n(\frac{1}{2} - \varepsilon)$ for a small, positive ε.

This is joint work with Andrzej Grzesik, and Piotr Kuc.

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Bartłomiej Bychawski
Jagiellonian University

Characterizations of unstable circulant graphs

Tensor product of graphs $\Gamma_1 = (V_1, E_1)$ and $\Gamma_2 = (V_2, E_2)$, denoted by $\Gamma_1 \times \Gamma_2$, is a graph with a vertex set $V_{\text{prod}} = V_1 \times V_2$ and edge set $E_{\text{prod}} = \{((v_1, v_2), (w_1, w_2)) \in V_{\text{prod}} \times V_{\text{prod}} \mid (v_1, w_1) \in E_1 \text{ and } (v_2, w_2) \in E_2\}$.

Natural question to ask is how automorphism group of $\Gamma_1 \times \Gamma_2$ look like? It is known that $\mathrm{Aut}(\Gamma_1) \times \mathrm{Aut}(\Gamma_2) \leq \mathrm{Aut}(\Gamma_1 \times \Gamma_2)$, but those groups do not have to be equal. Automorphism group of a tensor product is fully understood for connected and non-bipartite graphs thanks to Dörfler. On the other hand, if we multiply some connected and non-bipartite graph Γ even by K_2, it is not known in general how to decide whether $\mathrm{Aut}(\Gamma) \times \mathrm{Aut}(K_2)$ is the full automorphism group of $\Gamma \times K_2$ or not. We call a graph Γ *stable* if $\mathrm{Aut}(\Gamma) \times S_2 = \mathrm{Aut}(\Gamma \times K_2)$ and *unstable* otherwise.

We restrict ourselves to circulants, which are graphs whose automorphism group contains cyclic subgroup acting regularly on vertices. First important result for circulants was proved by B. Fernandez and A. Hujdurović in [1] and it characterized unstable circulants of odd order. Lately A. Hujdurović with I. Kovács [2] proved a characterization of unstable circulants of order $2p^e$ for any odd prime number p and $e \geq 1$. By applying methods involving classification of primitive group actions and cohomology of group modules, with Jakub Byszewski we extended their approach and derived classification of unstable circulants of squarefree order and of order $2pqr$, where p, q and r are not necessarily distinct odd primes.

This is joint work with Jakub Byszewski.

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Danila Cherkashin
Bulgarian Academy of Sciences

On set systems without singleton intersections

Let us define *Johnson graph* $J(n, k, t)$, whose vertices are k-element subsets of an n-element set and edges connect pairs of vertices with intersection t. An *independent set* is a vertex subset of a graph such that it does not contain an edge. Let $\alpha(G)$ be the size of a maximal independent set in a graph G.

In this language the statement of the Erdős–Ko–Rado theorem [1] is

$$\alpha(J[n, k, 0]) = \binom{n-1}{k-1}$$

for $n \geq 2k$. The problem of finding $\alpha(J[n, k, t])$ is known as Erdős–Sós forbidden intersection problem. The bibliography on this problem is wide and the proofs use very different techniques, see for instance [2, 3, 4].

We prove that for every $k > 1$ one has

$$\alpha(J[k^2 - k + 1, k, 1]) = \binom{k^2 - k - 1}{k - 2}.$$

The proof uses Hoffman bound applied to Johnson association scheme.

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Sylwia Cichacz

AGH University

Disjoint zero-sum sets in Abelian groups

A subset S of Γ is called a *zero-sum subset* if $\sum_{a \in S} a = 0$. The concept of disjoint zero-sum subsets in cyclic groups was inspired by Steiner triples research and started by Skolem [1]. It was later generalized for Abelian groups by Tannenbaum [2].

In this talk, we will present the recent progress of this topic as well as some applications of it in the irregular labeling of digraphs.

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Asaf Cohen Antonir
Tel Aviv University

On the Upper Tail Problem for irregular graphs

For a graph H let X_H denote the random variable counting the number of (unlabeled) copies of H in $G_{n,p}$. The study of this random variable goes back to the early works of Erdős and Rényi and its typical deviations are well understood. In this talk, we will discuss the large deviations of X_H. In particular, we will be interested in the upper tail problem, that is – what is the upper tail probability of H, namely, $\mathbb{P}(X_H > (1+\delta)\mathbb{E}[X_H])$?

The upper tail problem proved to be very difficult, and even when H is the triangle, the asymptotics of the logarithm of the upper tail probability, was solved only quite recently, by a breakthrough due to Harel, Mousset and Samotij [1]. More generally, together with a follow up result of Basak and Basu [2], the asymptotic value of the logarithm of the upper tail probability was resolved for any regular graph H.

Throughout the talk, we will first discuss the difficulties in estimating the upper tail probability for irregular graphs. Then, we will present an improvement for the state of the art theorem of Cook, Dembo and Pham [3], by estimating the logarithm of the upper tail probability of any irregular graph H, for sparser values of p, some of which are optimal.

The talk will be based on a joint work with Matan Harel, Frank Mousset and Wojciech Samotij.

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Sebastian Czerwiński
University of Zielona Góra

The Graham–Pollak theorem for bipartite graphs

The Graham–Pollak theorem says that the edges of a complete graph on n vertices cannot be partitioned into fewer than $n-1$ complete bipartite graphs. We can also partition a bicyclic $G=(V,H)$ graph into bicyclic graphs. If the partition P is proper, i.e. one bicyclic from P cannot cover one of the independent sets V or W, then Alon, Bohman, Holzman, and Kleitman proved that the partition contains at least 4 bipartite graphs. We present some results and conjectures about the partition P of a bicyclic $G=(V,W)$ into bicyclic such that less than k bicyclic from the partition P cannot cover one of the independent sets V or W.

Maciej Dołęga

Polish Academy of Sciences

Overview of map enumeration

In this talk, I will provide an overview of the enumeration of combinatorial maps, which are graphs embedded on surfaces. We'll begin with classical results by Tutte, then explore bijective methods, and finally, discuss the most recent developments based on topological recursion and integrable hierarchies.

Andrzej Dudek

Western Michigan University

The generalized Ramsey number $f(n, 5, 8) = \frac{6}{7}n + o(n)$

A (p, q)-*coloring* of K_n is a coloring of the edges of K_n such that every p-clique has at least q distinct colors among its edges. The generalized Ramsey number $f(n, p, q)$ is the minimum number of colors such that K_n has a (p, q)-coloring. Gomez-Leos, Heath, Parker, Schweider and Zerbib recently proved $f(n, 5, 8) \geq \frac{6}{7}(n - 1)$. In this talk, we show an asymptotically matching upper bound. Our construction is based on a randomized process, which we analyze by using "black box" theorems developed by two teams of researchers: (1) Glock, Joos, Kim, Kühn and Lichev, and (2) Delcourt and Postle.

This is joint work with Patrick Bennett and Ryan Cushman.

Bartłomiej Dudek

University of Wrocław

Finding the common core of some combinatorial problems

For many combinatorial problems we do know polynomial time algorithms, but proving their unconditional optimality remains a challenge, despite the problems have been studied for decades. In response, recently a new field called fine-grained complexity emerged, which focuses on establishing reductions between problems within P. This approach allows us to set conditional lower bounds, based on the difficulty of certain core problems.

Surprisingly, we can find fine-grained reductions between problems that seem unrelated, by uncovering their shared underlying structures. In this talk I will show examples of such reductions and present how some counting problems on graphs, trees and sequences are in fact equivalent. This not only leads to more efficient algorithms but also helps us state conditional lower bounds for these problems.

This is joint work with Paweł Gawrychowski and Tatiana Starikovskaya.

Anna Flaszczyńska
AGH University

Distinguishing vertices of graphs by sequences

In the paper [1] the authors distinguish vertices of a graph by sequences. This talk is about distinguishing vertices of a hypercube by sequences. Let f be the edge coloring of an n-dimensional hypercube. In a hypercube, we can define the order of edges, which results from the structure of this graph. Next, we can assign a sequence of colors to each vertex in such a way that the i-th element of this sequence is the color of the i-th edge coming from this vertex. We want to find a minimum number of colors to distinguish each pair of vertices in an n-dimensional hypercube.

This is joint work with Aleksandra Gorzkowska, and Mariusz Woźniak.

References

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Ugo Giocanti

Jagiellonian University

Coarse geometry of quasi-transitive graphs

Coarse geometry consists in studying the large-scale structure of a space. A central concept is the notion of *quasi-isometry*: intuitively, two metric spaces are quasi-isometric to each other if their metrics are the same, up to a rescaling by some constant factor. In particular, notions from coarse geometry turn out to be particularly relevant when studying the geometric properties of finitely generated groups, as every two different Cayley graphs of a given finitely generated group are quasi-isometric to each other.

Recently, a new area of research emerged that aims at studying the interplay between coarse geometry and graph theory, called *Coarse graph theory*. In this talk, I will focus on *quasi-transitive* graphs, i.e., graphs which have only a finite number of vertices, up to the application of an automorphism. Intuitively, quasi-transitive graphs have to be thought as graphs with "many symmetries", and this class of graphs is particularly interesting to study as it generalizes the class of vertex-transitive graphs and the class of Cayley graphs. In particular, we will see that every quasi-transitive locally finite graph which excludes a minor is quasi-isometric to some planar graph, and that every locally finite graph which is quasi-isometric to some planar graph is k-planar for some $k \in \mathbb{N}$. We will also discuss about a few other related recent questions and results.

This is joint work with Louis Esperet.

Przemysław Gordinowicz

Łódź University of Technology

From binary search through games to graphs

The talk focuses on some generalisations of binary search in a multidimensional or graph environment. They can be viewed as a game between the Adversary, who hides the target in this environment and responds to the Algorithm, which in turn, tries to find the target minimizing the number of queries.

We will provide some bounds for the number of queries, including bounds for random graphs and discuss the complexity of the optimal strategy for the Algorithm.

This is joint work with Dariusz Dereniowski (Gdańsk), Paweł Prałat (Toronto) and Karolina Wróbel (Łódź).

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Aleksandra Gorzkowska

AGH University

Arc-distinguishing of orientations of graphs

A distinguishing index of a graph is the minimum number of colours in an edge colouring such that the identity is the only automorphism that preserves the colouring. The study of the distinguishing index was started by Kalinowski and Pilśniak [12] and since then, there have been a number of results on the subject. In particular, the optimal bounds for the distinguishing index have been found for the classes of traceable or claw-free graphs. Recently, the variant of the problem for digraphs has attracted some interest. A distinguishing index of a digraph is the minimum number of colours in an arc colouring that is preserved only by the identity.

Meslem and Sopena [3] started a study of determining the minimum and maximum value of distinguishing index among all possible orientations of a given graph G. We continue this direction of investigation. However, we take a different approach to the problem and consider the relation between the distinguishing index of the orientations of G and the distinguishing index of G. In the talk, we present sharp results for trees, unbalanced bipartite graphs, traceable graphs and claw-free graphs. With this, we extend the results of Meslem and Sopena to some wider classes of graphs and answer a question posed by them about the class of complete bipartite graphs.

This is joint work with Jakub Kwaśny.

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Catherine Greenhill
University of New South Wales

Asymptotic enumeration, hypergraphs and directed hypergraphs

Asymptotic enumeration involves finding an approximate formula for the size of a combinatorial set, with a relative error that gets smaller as the size of the set grows. For example, we might be interested in the number of hypergraphs with a given number of vertices and satisfying some other nice properties. After discussing asymptotic enumeration results for hypergraphs, I will describe some recent joint work with Tamás Makai on asymptotically enumerating sparse directed hypergraphs.

Hubert Grochowski
Warsaw University of Technology

Approximation algorithms for $L(2,1)$-labeling of unit disk graphs

The $L(2,1)$-labeling of a graph is an assignment of non-negative integers to the vertices such that labels of adjacent vertices differ by at least 2, and labels of vertices at a distance of two from each other are distinct. This model is inspired by the frequency assignment problem in radio networks, so a natural and interesting class of graphs are disk intersection graphs, especially unit disk graphs.

Ono and Yamanaka proposed an 8-approximation algorithm for $L(2,1)$-labeling of unit disk graphs (see [1]). Their algorithm bases on some coloring of the Euclidean plane, called *shaded coloring*. Inspired by their work, we have introduced a family of approximation algorithms for $L(2,1)$-labeling of unit disk graphs, which base on a fractional version of shaded coloring. The best of our algorithms achieves an asymptotic approximation ratio less than 6.

This is joint work with Konstanty Junosza-Szaniawski.

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Igor Grzelec

AGH University

On some conjectures about local irregularity of graphs

We say that a graph is *locally irregular* if adjacent vertices have different degrees. After a short introduction about the Local Irregularity Conjecture [2] and updated version of this conjecture proposed by Sedlar and Škrekovski in [4] we present the Local Irregularity Conjecture for 2-multigraphs [3] with some already known and new results which supports this conjecture. Then we introduce the Standard (2,2) Conjecture [1] and the Weak (2,2) Conjecture [1] with some results about these two conjectures. At the end we present the Strong (2,2) Conjecture [1] and provide further new results which confirms this conjecture for subsequent graph classes.

This is joint work with Olivier Baudon, Julien Bensmail, Morgan Boivin, Clara Marcille, Alfréd Onderko, Arnaud Pêcher and Mariusz Woźniak.

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Andrzej Grzesik
Jagiellonian University

Subgraphs with a large minimum semidegree

A well known result of Erdős from the 60's states that every n-vertex undirected graph with m edges contains a subgraph with minimum degree at least $\frac{m}{n}$. This fact is used in many proofs in extremal graph theory to change an assumption on the number of edges of a graph to an assumption on the minimum degree of its subgraph, which is very useful in finding wanted substructures.

In the setting of directed graphs the number of edges is not an effective assumption for finding substructures – for example a transitive tournament has many edges but contains only acyclic subgraphs. A meaningful and often considered assumption for finding substructures in directed graphs is on the minimum outdegree. Since finding a given substructure is easier if we have not only an assumption on the minimum outdegree, but also an assumption on the minimum indegree, it is reasonable to ask if every directed graph with a large outdegree must contain a subgraph with a large minimum semidegree.

In the talk we show that every n-vertex directed graph with the minimum outdegree d contains a subgraph with minimum semidegree at least $\frac{d(d+1)}{2n}$. We also show that if $d = o(n)$ then this bound is asymptotically best possible.

This is joint work with Vojtěch Rödl and Jan Volec.

Yangyan Gu
Zhejiang Normal University

The Alon-Tarsi number of planar graphs revisited

We give a simple proof of the result that every planar graph G has Alon-Tarsi number at most 5, and has a matching M such that $G - M$ has Alon-Tarsi number at most 4.

This is joint work with Xuding Zhu.

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Jędrzej Hodor

Jagiellonian University

Weak coloring numbers of minor-closed graph classes

Weak coloring numbers are a family of graph parameters studied extensively in structural and algorithmic graph theory. We study the growth rate of weak coloring numbers of graphs excluding a fixed graph as a minor. Van den Heuvel et al. [1] showed that for a fixed graph X, the maximum r-th weak coloring number of X-minor-free graphs is polynomial in r. We determine this polynomial up to a factor of $\mathcal{O}(r \log r)$ and up to a factor of $\mathcal{O}(\log r)$ in a family of graphs of bounded treewidth. Moreover, we tie the exponent of the polynomial to a structural property of X, namely, 2-treedepth. Our result can be applied to improve several well-known bounds on weak coloring numbers. For instance, we show that for planar graphs of bounded treewidth, the maximum r-th weak coloring number is in $\mathcal{O}(r^2 \log r)$, which is best possible.

This is joint work with Hoang La, Piotr Micek, and Clément Rambaud.

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Nadzieja Hodur

AGH University

On 3-colourability of (*bull*, H)-free graphs

We call G an H-free graph, if G does not contain H as an induced subgraph. A *bull* is a triangle with two additional edges attached to its two vertices. In the class of *bull*-free graphs the 3-colourability problem remains NP-complete. However, in the class of graphs defined by two forbidden subgraphs, *bull* and one of stars $S(1,1,2)$ or $S(1,2,2)$, it is possible to find a polynomial algorithm that resolves 3-colourability. Such an algorithm returns a colouring if the given graph is 3-colourable, or a certain subgraph which is obviously non-3-colourable, otherwise.

In this talk we present such algorithms for (*bull*, $S(1,1,2)$)-free and (*bull*, $S(1,2,2)$)-free graphs. The main tool used is the characterisation of perfect graphs given by the Strong Perfect Graph Theorem.

This is joint work with Monika Pilśniak, Magdalena Prorok and Ingo Schiermeyer.

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Ilay Hoshen

Tel Aviv University

Stability of large cuts in random graphs

We prove that the family of largest cuts in the binomial random graph exhibits the following stability property: If $1/n \ll p \leq 1 - \Omega(1)$, then, with high probability, there is a set of $n - o(n)$ vertices that is partitioned in the same manner by all maximum cuts of $G_{n,p}$. Moreover, the analogous statement remains true when one replaces maximum cuts with nearly-maximum cuts.

We then demonstrate how one can use this statement as a tool for showing that certain properties of $G_{n,p}$ that hold in a fixed balanced cut hold simultaneously in all maximum cuts. In this talk, we provide two example applications of this tool. The first one shows that maximum cuts in $G_{n,p}$ typically partition the neighbourhood of every vertex into nearly equal parts; this resolves a conjecture of DeMarco and Kahn for all but a narrow range of densities p. We will also mention another application regarding sharp thresholds in Turán type problems.

This is joint work with Wojciech Samotij and Maksim Zhukovskii.

Justyna Jaworska

Jagiellonian University

Generalized Turán problem for directed cycles

In extremal graph theory, by $\mathrm{ex}(n, H, F)$, we denote the maximum number of copies of graph H that an n-vertex graph without any copies of F can contain. We study this quantity for oriented graphs when both H and F are directed cycles. Let $\overrightarrow{C_i}$ denote a directed cycle on i vertices. We establish the order of magnitude of $\mathrm{ex}(n, \overrightarrow{C_k}, \overrightarrow{C_\ell})$ for all pairs (k, ℓ) and also calculate the value up to a lower order error term when $k \ll \ell,\ \ \ell \nmid k$.

We then present partial results and conjectures in the remaining cases, which show a multitude of possible extremal constructions, quite uncommon for a problem with such a simple statement.

This is joint work with Andrzej Grzesik, Bartłomiej Kielak, Piotr Kuc and Tomasz Ślusarczyk.

Rafał Kalinowski

AGH University

The Unfriendly Partition Conjecture holds for line graphs

A colouring of edges of a graph G is a majority colouring, if for every vertex v of G, at most half the edges incident with v have the same colour. This concept was recently introduced in [1] where, among others, it was proved that every finite graph without pendant vertices admits a majority 4-edge colouring. Moreover, if the minimum degree of G is at least 4, then G admits a majority 3-edge colouring.

In the talk, the list version of the problem is investigated, also for infinite graphs. As a consequence of our results, the Unfriendly Partition Conjecture (cf. Diestel's book) is confirmed for line graphs.

This is joint work with Monika Pilśniak and Marcin Stawiski.

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Mateusz Kamyczura

AGH University

Conflict-free edge coloring of nearly regular graphs

Let G represent a graph characterized by a maximum degree Δ. A coloring c of the edges in graph G is classified as conflict-free when each edge's closed neighborhood contains a uniquely colored element. The minimum number of colors required for such a coloring, denoted as the conflict-free chromatic index of G and represented by $\chi'_{CF}(G)$. Utilizing the probabilistic method, Dębski and Przybyło [1] showed that certain graph families exist for which $\chi'_{CF}(G) \geq (1 - o(1)) \log_2 \Delta$. Later Kamyczura, Meszka, Przybyło [2] showed, that $\chi'_{CF}(G) \leq 3\lceil \log_2 \Delta \rceil + 1$, which was a direct consequence of a more robust assertion: $\chi'_{CF}(G) \leq 3\lceil \log_2 \chi(G) \rceil + 1$. We proved that for complete graphs $\chi'_{CF}(K_n) \leq \lceil \log_2(n-1) \rceil + 1$ and using probabilistic methods we proved that for the almost regular graphs with sufficiently large Δ, $\chi'_{CF}(G) \leq \log_2 \Delta(1 + o(1))$.

This is joint work with Jakub Przybyło.

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Aleksander Katan
Jagiellonian University

Majority colorings of countable dags

A *majority coloring* of a directed graph is an assignment of colors to vertices in which, for each vertex, at most half of its out-neighbors share its color. We present an encoding of boolean formulas into digraphs, such that in any majority 2-coloring of the digraph, the color of a specified output vertex is dependent only on the colors of input vertices and corresponds to the valuation of the formula. This encoding is used to show the following:

- a simple proof that deciding whether a digraph is majority 2-colorable is **NP**-complete – another way of obtaining the result proven in [1],
- an example of a countable acyclic digraph that is not majority 2-colorable, that was conjectured to not exist in [2], [3], and [4].

This is joint work with Bartłomiej Bosek.

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Peter Keevash
University of Oxford

Isoperimetric stability and its applications

We will survey some results on the stability of isoperimetric problems, broadly interpreted to include several directions in Combinatorics, Probability, Analysis, Geometry and Group Theory. The unifying theme of these stability results is describing the approximate structure implied by small growth. We will also discuss some applications to Extremal and Probabilistic Combinatorics and Group Theory. These results are joint work with various combinations of Barber, Erde, van Hintum, Lifshitz, Long, Minzer, Roberts and Tiba.

Tomasz Kościuszko
Adam Mickiewicz University

Sets with no solutions to some symmetric linear equations

In this talk we consider symmetric linear equations of the form

$$a_1x_1 + a_2x_2 + \cdots a_kx_k = a_1x'_1 + a_2x'_2 + \cdots a_kx'_k,$$

where $a_1, a_2, \cdots, a_k$ are integer coefficients and $x_1, x'_1, x_2, x'_2, \cdots$ are variables taking values in the subset A of the first N positive integers. It is expected that for every such equation there exists a subset A with $|A| \gg N^{1/k}$ which contains no non-trivial solutions to the equation. Ruzsa [1] showed how to construct such A for the case $k = 2$ and in general he noticed that the greedy construction always gives $|A| \gg N^{1/(2k-1)}$.

In this talk we present our new technique which gives near-optimal constructions for a range of equations, for instance when the coefficients are picked at random from the interval $[1, C]$, for a large C.

We analyze in what cases our technique does not work and we offer a partial resolution. For the case $k = 3$ we show that there exists a set with $|A| \gg N^{1/4.77}$ assuming that two out of three coefficients are large enough, roughly comparable and coprime.

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Julia Kozik
AGH University

Integrity of grids

The integrity of a graph $G = (V, E)$ is defined as the smallest sum $|S| + m(G - S)$, where S is a subset of the set V, and $m(H)$ denotes the order of the largest component of the graph H.

Benko, Ernst, and Lanphier provided and proved an asymptotic bounds for planar graphs in terms of the order of the graph. We prove asymptotic results concerning two-dimensional grid-graphs.

This is joint work with Andrzej Żak.

Tomasz Krawczyk

Warsaw University of Technology

Forbidden induced subgraphs for (Helly) circular-arc graphs

One of the main problems on a hereditary class of graphs is to characterize all minimal graphs that are not in this class (as induced subgraphs). For basic intersection graph classes (interval graphs, permutation graphs, function graphs, or chordal graphs) the list of all minimal forbidden graphs has been compiled, but for some (circular-arc graphs, circle graphs) it is still unknown.

In my talk I will present a method that allows us to identify all minimal graphs that are not circular-arc graphs within the class of chordal graphs. Surprisingly, such graphs turn out to have very simple structures: all the nontrivial ones belong to a single family. I will also discuss the obstacles that must be overcome to obtain a complete list of minimal graphs that are not circular-arc graphs.

This is a joint work with Yixin Cao.

Borys Kuca
Jagiellonian University

The Szemerédi theorem and beyond

Looking for patterns in sets of numbers is among the oldest and most fundamental mathematical endeavors. A quintessential result in this direction is the Szemerédi theorem which asserts that each subset of integers of positive density contains an arithmetic progression of arbitrary (finite) length. Often viewed as an example of "deep" mathematics due to its elaborate and diverse proofs, the Szemerédi theorem has stimulated far-reaching developments in areas as diverse as combinatorics, number theory, harmonic analysis, ergodic theory and model theory. In this talk, I will survey some of the recent progress on the Szemerédi theorem and its generalisations.

Gaurav Kucheriya

Charles University

Orientations of graphs with at most one directed path between every pair of vertices

Given a graph G, we say that an orientation D of G is a *KT orientation* if, for all $u, v \in V(D)$, there is at most one directed path (in any direction) between u and v. Graphs that admit such orientations have been used by Kierstead and Trotter (1992), Carbonero, Hompe, Moore, and Spirkl (2023), Briański, Davies, and Walczak (2023), and Girão, Illingworth, Powierski, Savery, Scott, Tamitegami, and Tan (2024) to construct graphs with large chromatic number and small clique number that served as counterexamples to various conjectures.

Motivated by this, we consider which graphs admit KT orientations (named after Kierstead and Trotter). In particular, we construct a graph family with small independence number (sub-linear in the number of vertices) that admits a KT orientation. We show that the problem of determining whether a given graph admits a KT orientation is NP-complete, even if we restrict ourselves to planar graphs. Finally, we provide an algorithm to decide if a graph with maximum degree at most 3 admits a KT orientation. Whereas, for graphs with maximum degree 4, the problem remains NP-complete.

This is joint work with Barbora Dohnalová, Jiří Kalvoda, and Sophie Spirkl.

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Sandeep Kumar
South Asian University

On the Index of Tricyclic Signed Graphs

A *signed graph* Σ is an ordered pair (Σ^u,σ) that consists of a *underlying graph* Σ^u=(V,E) and a sign mapping called *signature* σ from E to the sign set $\{+,-\}$. The *negative section* of a signed graph Σ is the maximal connected edge-induced subgraph in Σ consisting of only the negative edges of Σ. Let $\mathcal{T}_n^p$ denote the set of all unbalanced tricyclic signed graphs of order $n \geq 4$, with p ($1 \leq p \leq 3$) unbalanced cycles. We have characterize the structure of the signed graphs achieving the maximal *index* (largest eigenvalue of adjacency matrix) among all signed graphs in $\mathcal{T}_n^p$. We also establish that if Σ is a connected signed graph with fixed number of vertices, positive edges, negative edges and maximizes the *index* then the induced subgraph of negative edges forms a *negative section* and the Laplacian spectrum of that induced subgraph will have exactly one zero.

This is joint work with Deepa Sinha.

Eden Kuperwasser

Tel-Aviv University

On the Kohayakawa–Kreuter conjecture

We say that G is Ramsey for a tuple of graphs $(H_1, \dots, H_r)$ if any r-coloring of the edges of G contains, for some $i \in [r]$, a monochromatic copy of H_i in the color i. In their seminal work, Rödl and Ruciński [3] located the threshold at which the binomial random graph $G_{n,p}$ becomes Ramsey for the symmetric case where all H_i are identical. Building on this work, Kohayakawa and Kreuter [2] conjectured where the threshold for the general case should be.

The subject of this talk is a joint work with Wojciech Samotij and Yuval Wigderson, in which we resolve almost all cases of the conjecture. We do so by reducing the conjecture to a deterministic statement, which we then prove for most cases.

In a recent subsequent work of Christoph, Martinsson, Steiner, and Wigderson [1], this deterministic statement was proven in full generality, thus proving the conjecture.

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Jakub Kwaśny
AGH University

Distinguishing regular graphs

An edge colouring of a graph is called distinguishing if there is no nontrivial automorphism which preserves it. We study this type of colouring on a class of connected regular graphs. We improve the result of Lehner, Pilśniak, and Stawiski, which states that there is a distinguishing 3-edge-colouring of any connected regular graph except K_2. Namely, we prove that every at most countable, finite or infinite, connected regular graph of order at least 7 admits a distinguishing edge colouring from any set of lists of length 2. Furthermore, we show that the same holds for connected regular graphs of order κ where κ is a fixed point of the aleph hierarchy.

This is joint work with Marcin Stawiski.

Hoang La

GALAC, LISN, University Paris-Saclay

Fractional domatic number and minimum degree

The domatic number of a graph G is the maximum number of pairwise disjoint dominating sets of G. We are interested in the LP-relaxation of that parameter, which is called the fractional domatic number of G. We study its extremal value in the class of graphs of minimum degree d, and show that it is always at least $(1 - o(1))\, d/\ln d$ as $d \to \infty$. This is asymptotically tight as certified by a study from 1990 of dominating sets in random graphs by Alon [1]; we exhibit a deterministic construction that also reaches this asymptotic bound. We then focus on the case $d = 2$, and show that, except for finitely many exceptional graphs, the fractional domatic number of a connected graph of minimum degree (at least) 2 is at least $5/2$. This proves in a stronger sense a conjecture by Gadouleau, Harms, Mertzios, and Zamaraev [2]. This also extends and generalises results from McCuaig and Shepherd [3], from Fujita, Kameda, and Yamashita [4], and from Abbas, Egerstedt, Liu, Thomas, and Whalen [5]. Finally, we show that planar graphs of minimum degree at least 2 and girth at least g have fractional domatic number at least $3 - O(1/g)$ as $g \to \infty$.

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Richard Lang
Polytechnic University of Catalonia

A hypergraph bandwidth theorem

A classic result of Dirac states that a graph has a Hamilton cycle provided that every vertex is adjacent to at least half of the other vertices. The Bandwidth Theorem guarantees that under mildly stronger assumptions one can even embed all bipartite graphs with sublinear bandwidth and constant maximum degree. Moreover, analogous statements hold for higher chromatic numbers. In this talk, I present an extension of this result to hypergraphs.

This is joint work with Nicolás Sanhueza-Matamala.

Clément Legrand-Duchesne
Jagiellonian University

A recoloring version of a conjecture of Reed

In [6], Reed conjectured that the chromatic number of any graph is closer to its clique number than to its maximum degree plus one, which are two trivial bounds on χ:

Conjecture 1 ([6]) *For all graph G,*

$$\chi(G) \leq \left\lceil \frac{\omega(G) + \Delta(G) + 1}{2} \right\rceil .$$

In particular, for any $\varepsilon \leq 1/2$ and any graph G,

$$\chi(G) \leq \lceil (1-\varepsilon)(\Delta(G)+1) + \varepsilon\omega(G) \rceil .$$

In the same article, Reed proved using the probabilistic method that this bound is tight if true. He also proved that there exists $\varepsilon > 0$ such that for all G, $\chi(G) \leq \lceil (1-\varepsilon)(\Delta(G)+1) + \varepsilon\omega(G) \rceil$.

This conjecture received a large interest over the past years [5, 2, 3], the best bound known today is due to Hurley, de Joannis de Verclos and Kang:

Theorem 2 ([4]) *For all graph G with sufficiently large maximum degree,*

$$\chi(G) \leq \lceil 0.881(\Delta(G)+1) + 0.119\omega(G) \rceil .$$

We consider here a recoloring version of this conjecture. A *Kempe change* is a reconfiguration operation on colorings that swaps two colors within a maximal bichromatic component. We say that two colorings are *Kempe equivalent* if they differ by a series of Kempe changes. A graph is *k-recolorable* if all its k-colorings are Kempe equivalent.

Question 1 *What is the largest $\varepsilon \leq 1/2$ such that all graphs G are k-recolorable for all $k \geq \lceil (1-\varepsilon)(\Delta(G)+1) + \varepsilon\omega(G) \rceil$?*

The main obstacle to k-recolorability are *frozen* colorings (which we will not define here). The largest ε for which there exists graphs with frozen $\lceil(1-\varepsilon)(\Delta+1)+\varepsilon\omega\rceil$-colorings is $\varepsilon = 1/3$ [1]. Hence ε is at most $1/3$ on general graphs and we conjecture that is the right answer to Question 1.

When restricting to odd-hole free graphs (that contain no induced odd-cycle), we show that the $\varepsilon = 1/2$:

Theorem 3 *All odd-hole free graphs k-recolorable, for $k \geq \left\lceil \frac{(\Delta+1)+\omega}{2} \right\rceil$.*

This is joint work with Lucas De Meyer, Tomaś Kaiser, Jared León, Youri Tamitegama and Tim Planken.

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Anna M. Limbach
Czech Academy of Sciences

Graphon branching processes and fractional isomorphism

In their study of the giant component in inhomogeneous random graphs, Bollobás, Janson, and Riordan introduced a class of branching processes parametrized by a possibly unbounded graphon. We prove that two such branching processes have the same distribution if and only if the corresponding graphons are fractionally isomorphic, a notion introduced by Grebík and Rocha.

This is joint work with Jan Hladký and Eng Keat Hng.

Mateusz Litka
Adam Mickiewicz University

Online size Ramsey number for C_4 and P_6

In this lecture we consider a game played on the edge set of the infinite clique $K_{\mathbb{N}}$ by two players, *Builder* and *Painter*. In each round of the game, Builder *chooses* an edge and Painter *colors* it red or blue. Builder wins when Painter creates a red copy of G or a blue copy of H, for some fixed graphs G and H. Builder wants to win in as few rounds as possible, and Painter wants to delay Builder for as many rounds as possible.

The online size Ramsey number $\tilde{r}(G, H)$, is the minimum number of rounds within which Builder can win, assuming both players play optimally.

So far it has been proven by Dybizbański, Dzido and Zakrzewska that $11 \leq \tilde{r}(C_4, P_6) \leq 13$ [1]. In this lecture, we refine this result and show the exact value, namely we will present the Theorem that $\tilde{r}(C_4, P_6) = 11$, with the details of the proof.

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Tomáš Masařík
University of Warsaw

Proving a directed analogue of the Gyárfás-Sumner conjecture for orientations of P_4

An oriented graph is a digraph that does not contain a directed cycle of length two. An (oriented) graph D is H-free if D does not contain H as an induced sub(di)graph. The Gyárfás-Sumner conjecture is a widely-open conjecture on simple graphs, which states that for any forest F, there is some function f such that every F-free graph G with clique number $\omega(G)$ has chromatic number at most $f(\omega(G))$. Aboulker, Charbit, and Naserasr [1] proposed an analogue of this conjecture to the dichromatic number of oriented graphs. The dichromatic number of a digraph D is the minimum number of colors required to color the vertex set of D so that no directed cycle in D is monochromatic.

Aboulker, Charbit, and Naserasr's $\overrightarrow{\chi}$-boundedness conjecture states that for every oriented forest F, there is some function f such that every F-free oriented graph D has dichromatic number at most $f(\omega(D))$, where $\omega(D)$ is the size of a maximum clique in the graph underlying D. In our paper [2], we perform the first step towards proving Aboulker, Charbit, and Naserasr's $\overrightarrow{\chi}$-boundedness conjecture by showing that it holds when F is any orientation of a path on four vertices.

This is joint work with Linda Cook, Marcin Pilipczuk, Amadeus Reinald, and Uéverton S. Souza.

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Mariusz Meszka
AGH University

Two-factorizations of some regular graphs

A *k-factor* in a graph G is a k-regular spanning subgraph of G. A *k-factorization* of G is a collection $\{F_1, F_2, \ldots, F_t\}$ of edge-disjoint k-factors such each edge of G belongs to exactly one F_i. We say that G has an *F-factorization* if each F_i, $i = 1, 2, \ldots, t$, is isomorphic to F.

One of the best-known open problems concerning two-factorizations is the famous Oberwolfach problem, posed by G. Ringel in 1967, which asks whether, for any two-factor F, the complete graph K_n (when n is odd) or $K_n \setminus I$ (when n is even and I is a one-factor removed from K_n) admits an F-factorization. Several years later A. Rosa suggested the following extension of the Oberwolfach problem, the so-called Hamilton–Waterloo problem, which asks for the existence of a two-factorization of K_n or $K_n \setminus I$ (depending on the parity of n) in which r of its two-factors are isomorphic to a given two-factor R, and the remaining q two-factors are isomorphic to a given two-factor Q, for any admissible r and q.

Results related to both these problems will be presented. Moreover, algorithmic methods for constructing two-factorizations will be discussed.

Piotr Micek

Jagiellonian University

Planar graphs in blowups of fans

We show that every n-vertex planar graph is contained in the graph obtained from a fan by blowing up each vertex by a complete graph of order $O(\sqrt{n}\log^2 n)$. Equivalently, every n-vertex planar graph G has a set X of $O(\sqrt{n}\log^2 n)$ vertices such that $G - X$ has bandwidth $O(\sqrt{n}\log^2 n)$. This result holds in the more general setting of graphs contained in the strong product of a bounded treewidth graph and a path, which includes bounded genus graphs, graphs excluding a fixed apex graph as a minor, and k-planar graphs for fixed k. These results are obtained using two ingredients. The first is a new local sparsification lemma, which shows that every n-vertex planar graph G has a set of $O((n \log n)/D)$ vertices whose removal results in a graph with local density at most D. The second is a generalization of a method of Feige and Rao, that relates bandwidth and local density using volume-preserving Euclidean embeddings.

This is joint work with Vida Dujmović, Gwenaël Joret, Pat Morin, and David R. Wood.

Wojciech Michalczuk

Wrocław University of Science and Technology

Normal approximation for subgraph count in random hypergraphs

A non-uniform and inhomogeneous random hypergraph model is considered, which is a natural extension of the well-studied binomial Erdös-Rényi random graph model $\mathbb{G}(n,p)$. Let $\mathbb{H}(n,\mathbf{p})$, $\mathbf{p}(n) = \big(p_1(n),...,p_n(n)\big)$, be a generalization of $\mathbb{G}(n,p)$ to hypergraphs, where any hyperedge of size r appears, independently of other edges, with probability $p_r(n)$. For a fixed hypergraph H we denote by N_n^H the number of subhypergraphs of $\mathbb{H}(n,\mathbf{p})$ that are isomorphic to a fixed hypergraph H. The problem we examine is when and how fast its normalization $\widetilde{N}_n^H$ converges to the standard normal distribution $\mathcal{N}(0,1)$. While the binomial $\mathbb{G}(n,p)$ model has been extensively studied with asymptotic normality for the graph version of $\widetilde{N}_n^H$ characterized in [3] and convergence rates obtained in [1, 2], literature on general, non-uniform and inhomogeneous, random hypergraphs remains modest.

In the talk, we present a necessary and sufficient condition for small hypergraphs count N_n^H to be asymptotically normal, complemented with convergence rate in both the Wasserstein and Kolmogorov distances. Next, we focus on the homogeneous model to relate the obtained results to the so-called *fourth moment phenomenon.* Then, we provide an intriguing trick that proves the necessity of the equivalence condition within the homogeneous framework, particularly in the model $\mathbb{G}(n,p)$.

This is joint work with Mikołaj Nieradko and Grzegorz Serafin [4].

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Mirjana Mikalački

University of Novi Sad

The burning game

The virus propagation in both medicine and in computer science or spreading the influence over social media are just some examples of natural and engineered phenomena spread over the networks, that are active research topics. In all these examples the key question is how quickly can the contagion spread over all the members in the network. To address this question, as a simplified deterministic model, the process of *burning* of the graph was introduced in [1]. Motivated by COVID-19 pandemics, in order to slow down the burning process, with aim to reduce the speed of spreading the infections, and virus propagation, the *cooling process* of a graph was introduced recently in [2].

Motivated by these two processes, we introduce the new graph game, *the burning game*, played by two players, *Burner* and *Staller*. Given a graph, vertices can be burned or unburned, but once burned, they stay in this state until the end of the game. The players alternate in choosing the previously unburned vertices of a given graph, with different aims: Burner wants to burn the graph as quickly as possible, and Staller wants to prolong this process as much as possible. At time step $t = 0$, all vertices are unburned. In each time step $t \geq 1$, first all neighbours of burned vertices become burned, and then one of the players burns one unburned vertex in this time step as well. The game ends in the first time step t in which all vertices of G are burned.

We analyse the burning number, i.e. the minimum number of steps required for the graph to burn, give some basic properties and characterizations.

This is joint work with Nina Chiarelli, Vesna Iršič, Marko Jakovac and Bill Kinnersley.

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Patryk Morawski

ETH Zurich

Oriented Ramsey numbers of bounded degree digraphs

While Ramsey numbers can grow exponentially with the number of vertices in a graph, the Burr-Erdős conjecture, resolved by Lee [1], establishes that the Ramsey number of any graph with maximum degree Δ is bounded above by $c_\Delta |V(G)|$, where c_Δ is a constant dependent on Δ. However, quite surprisingly, Fox, He, and Wigderson [2] have recently shown linear bound does not hold when considering oriented graphs, where edges have a direction. The question remains open of whether bounded-degree oriented graphs possess Ramsey numbers that are at most polynomial in the number of vertices.

In this talk, we will present our findings for a particular class of oriented graphs known as graded digraphs. We demonstrate that every graded digraph with maximum degree Δ has an oriented Ramsey number bounded by $c^\Delta |V(G)|$ for some absolute constant c.

Furthermore, we explore the related problem of determining upper bounds on the oriented Ramsey numbers of a digraph based on the number of its edges. We prove that for any oriented digraph G with no isolated vertices, the oriented Ramsey number is at most $2^{c\sqrt{|E(G)|}\log\log|E(G)|}$, where c is an absolute constant.

This is joint work with Yuval Wigderson, Benny Sudakov and Domagoj Bradač.

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Bogdana Oliynyk
Silesian University of Technology

The unitary Cayley graph of upper triangular matrix rings

The unitary Cayley graph C_R of a finite unital ring R is the simple graph with vertex set R in which two elements x and y are adjacent if and only if $x-y$ is a unit of R. First, the unitary Cayley graphs of rings were considered by Dejter and Giudici [1] for the rings $\mathbb{Z}_n$.

We characterize the unitary Cayley graph $C_{T_n(\mathbb{F})}$ of the ring of all upper triangular matrices $T_n(\mathbb{F})$ over a finite field $\mathbb{F}$. We show that $C_{T_n(\mathbb{F})}$ is isomorphic to the semistrong product of the complete graph K_m and the antipodal graph of the Hamming graph $A(H(n, |\mathbb{F}|))$ for some m. In particular, if the field F contains only two elements, the graph $C_{T_n(\mathbb{F})}$ has 2^{n-1} connected components, each of them is isomorphic to the complete bipartite graph. We also characterize the clique number and the chromatic number and describe the tight upper and the tight lower bounds for the domination number of $C_{T_n(\mathbb{F})}$.

This is joint work with Waldemar Hołubowski, Sergiy Kozerenko, and Viktoriia Solomko.

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Paweł Pękała
AGH University

List variants of majority edge-colourings of graphs

A *majority edge-colouring* of a graph G is a colouring of the edges of G such that for each vertex v of G, at most half the edges incident with v have the same colour. More generally, for a natural number $k \geq 2$, a $1/k$*-majority edge-colouring* of a graph is a colouring of the edges of G such that for every colour i and every vertex v of G at most $1/k$ of the edges incident with v have the colour i. This notion was introduced recently by Bock, Kalinowski, Pardey, Pilśniak, Rautenbach and Woźniak [1].

We consider the list version of $1/k$-majority edge-colourings. In particular, we provide an upper bound on the minimum degree of a graph which necessitates the existence of a $1/k$-majority edge-colouring from lists of size $k+1$. In addition, we discuss some further generalizations of $1/k$-majority edge-colourings.

This is joint work with Jakub Przybyło.

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Marta Piecyk

Warsaw University of Technology

HOMOMORPHISMS OF BOUNDED-DIAMETER GRAPHS

For a fixed graph H, in the graph homomorphism problem, denoted by HOM(H), we are given a graph G and we have to determine whether there is an edge preserving mapping $\varphi : V(G) \to V(H)$. Note that HOM(C_3), where C_3 is the cycle of length 3, is equivalent to 3-COLORING. The question of whether 3-COLORING is polynomial-time solvable on diameter-2 graphs is a well-known open problem.

In my talk I will consider HOM(C_{2k+1}) for $k \geq 2$, i.e., all other odd cycles as target graphs. I will show that for $k \geq 2$, the HOM(C_{2k+1}) problem is polynomial-time solvable on diameter-$(k+1)$ graphs – note that such a result for $k = 1$ would be precisely a polynomial-time algorithm for 3-COLORING of diameter-2 graphs. I will also show some other related results.

Magdalena Prorok

AGH University

Rainbow Turán problems

One of the central topics in extremal graph theory, known as the Turán problem, is to determine the maximum number of edges of a graph on n vertices that does not contain a copy of a given graph F as a subgraph. Equivalently, the minimum number of edges that forces the existence of F as a subgraph.

In a rainbow version of this problem, for an integer $c \geq 1$ we consider a collection of c graphs $\mathcal{G} = (G_1, \ldots, G_c)$ on a common vertex set, thinking of each graph as edges in a distinct color. We want to force the existence of a rainbow copy of F in $\mathcal{G}$ by having a large number of edges in each graph.

In this talk we present a solution to the problem for directed graphs without rainbow triangles and stars for any number of colors.

Based on joint works with Sebastian Babiński, Dániel Gerbner, Andrzej Grzesik and Cory Palmer.

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Arsenii Sagdeev

Karlsruhe Institute of Technology

Canonical theorems in Euclidean Ramsey theory

We prove the following two results in Euclidean Ramsey theory. First, every coloring of the space, regardless of the number of colors used, contains either a monochromatic or a rainbow congruent copy of each acute triangle. Second, every coloring of $\mathbb{R}^n$ contains either a monochromatic or a rainbow congruent copy of an m-dimensional unit hypercube, provided that n is sufficiently large in terms of m.

This is joint work with Panna Gehér and Géza Tóth.

Giovanne Santos

University of Chile

Antidirected trees in directed graphs

Given two graphs H and G, what condition on the minimum degree of G guarantees that H appears as a subgraph of G? A fundamental result of Dirac from 1952 [1] implies that any graph on $n \geq 3$ vertices of minimum degree at least $n/2$ contains a Hamilton cycle as a subgraph. For directed graphs G we can ask instead for conditions on the minimum semidegree of G, which is defined as the minimum over all the in- and all the out-degrees of the vertices. Recently, Stein and Zárate-Guerén [2] proved that every sufficiently large oriented graph (a directed graph without multiple edges) on n vertices with minimum semidegree $k/2 + o(n)$ contains every balanced antidirected tree with k edges and bounded maximum degree.

It is natural to try to extend Stein and Zárate-Guerén's result for all directed graphs. However, this would not even be true if we replace the lower bound on the minimum semidegree by $k - 1$, since the disjoint union of two complete digraphs on k vertices does not contain any tree on k edges. In this talk, we will show that if we add a condition that G has a vertex of high in- and out-degree, a minimum semidegree of $3k/5+o(n)$ guarantees the existence of any balanced antidirected tree on k edges and bounded maximum degree in G.

This is joint work with George Kontogeorgiou, and Maya Stein.

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Grzegorz Serafin

Wrocław University of Science and Technology

Small subgraphs count in random intersection graph

Asymptotic behaviour of small subgraphs count is a problem with a very long history in the theory of random graphs. While a lot in known in the case of the classical binomial model $\mathbb{G}(n,p)$, other models seem not that easy to approach.

In this talk, we address this problem in the context of the random intersection graph model $\mathbb{G}(n,m,p)$. Here, the threshold function for subgraph containment is known and the distribution of the number of copies on the threshold has been already determined. However, the normal approximation of the number of copies of small subgraphs remains open. Recently only a partial result on triangles count has appeared.

We propose a new method that allows us effectively deal with number of edges and triangles, and might be adapted to other cliques as well. We establish necessary and sufficient conditions for the asymptotic normality and complement them with corresponding convergence rates in both the Wasserstein and the Kolmogorov distances.

Matas Šileikis

Czech Academy of Sciences

Maximum local tree counts in the random binomial graph

Maximum degree of the random binomial graph $G(n, p)$ was thoroughly studied by Ivchenko [1] and Bollobás [2]. We consider the following generalization of the maximum degree. For a fixed rooted tree T, let X_v be the number of copies (injective homomorphisms) of T into $G(n, p)$ such that the root of T is mapped to a vertex v. This is also a special case of so-called extension counts, introduced by Spencer [3]. Analogous maximum clique counts were studied in [4].

We ask the question where the maximum of X_v over all vertices is concentrated. For p tending to zero not too fast, we show that the maximum is asymptotically attained by the vertex of maximum degree. That is, if the maximum degree is concentrated at value $D = D(n, p)$, then the maximum T-count is concentrated at $D^d (pn)^{e(T)-d}$, where d is the degree of the root in T. However, for smaller p, the maximum is concentrated at a value of higher order and the answer crucially depends on the structure of T. We illustrate this by treating fully the case when T is a path rooted at the end.

This is joint work with Pedro Araújo (CTU in Prague), Simon Griffiths (PUC Rio) and Lutz Warnke (UCSD).

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Marek Skrzypczyk

Jagiellonian University

Quadratic embedding of graphs

Let $G = (V, E)$ be a connected graph. A *quadratic embedding* is a map $\varphi : V \to \mathcal{H}$ satisfying

$$\|\varphi(x) - \varphi(y)\|^2 = d(x, y), \quad x, y \in V,$$

where $(\mathcal{H}, \|\cdot\|)$ is some Hilbert space and $d(x, y)$ is the graph distance.

A graph G is said to be *of QE class* (*of non-QE class*) if it admits a quadratic embedding (does not admit a quadratic embedding, respectively). Moreover, a graph of non-QE class is called *primary* if it contains no isometrically embedded proper subgraphs of non-QE class.

Obata showed all primary non-QE graphs on 5 and 6 vertices, as well as all complete multipartite primary non-QE graphs. The main problem is to find all primary non-QE graphs and whether they exist for each number of vertices greater than or equal 5. We provide a positive answer to this question.

This is joint work with Wojciech Młotkowski and Michał Wojtylak.

Marek Sokołowski

University of Warsaw

Combinatorics of graphs of bounded twin-width

Twin-width is a graph parameter assigning to every undirected graph a non-negative integer measuring the similarity of the graph to a cograph. This notion was introduced by Bonnet, Kim, Thomassé and Watrigant [J. ACM 2021], who adapted and generalized a similar width parameter used in the study of permutations. Their work quickly gained attention due to the deep algorithmic, combinatorial and logical properties of classes of graphs of bounded twin-width.

In this talk, I will present twin-width and some of our recent results related to the combinatorics of this graph parameter, in particular how to encode graphs of small twin-width succinctly and how to estimate the chromatic number of such graphs.

Marcin Stawiski

AGH University

Distinguishing graphs by list edge-colourings

We say that an edge colouring *breaks* an automorphism if some edge is mapped to an edge of a different colour. We say that the colouring is *distinguishing* if it breaks every non-identity automorphism. We show that such a colouring can be chosen from any set of lists associated to the edges of a graph G, whenever the size of each list is at least $\Delta - 1$, where Δ is the maximum degree of G, apart from some classified exceptions. This holds both for finite and infinite graphs. The bound is optimal for every Δ, and it is the same as in the non-list version.

This is joint work with Jakub Kwaśny.

Miloš Stojaković
University of Novi Sad

Moving and stacking squares for greater visibility

We consider unit square symbols that need to be placed at specified y-coordinates. Our hope is to optimize the drawing order of the symbols as well as their x-displacement, constrained within a rectangular container, to maximize the minimum visible perimeter over all squares.

If the container has width and height at most 2, there is a point that stabs all squares. In this case, we prove that a staircase layout is arbitrarily close to optimality and can be computed in $O(n \log n)$ time. If the width is at most 2, there is a vertical line that stabs all squares, and in this case we give a 2-approximation algorithm (assuming fixed container height) that runs in $O(n \log n)$ time. As a minimum visible perimeter of 2 is always trivially achievable, we measure this approximation with respect to the visible perimeter exceeding 2. We show that, despite its simplicity, the algorithm gives asymptotically optimal results for certain instances.

This is joint work with Bernd Gärtner, Vishwas Kalani, Meghana M. Reddy, Wouter Meulemans and Bettina Speckmann.

John Sylvester

University of Liverpool

Recent progress on adjacency labelling schemes

An adjacency labeling scheme for a *hereditary* (closed under induced subgraphs) class $\mathcal{C}$ of graphs defines, for any n-vertex $G \in \mathcal{C}$, an assignment of labels to each vertex in G so that adjacency in G is determined by a (fixed) function of the two labels of the endpoints. By a counting argument, if there are $|\mathcal{C}_n|$ many n-vertex graphs in $\mathcal{C}$, then any adjacency labeling scheme needs labels with at least $(\log |\mathcal{C}_n|)/n$ many bits. If such a scheme exists (upto a constant), then it is called an implicit representation.

Many classes of graphs admit an implicit representation and in 1988 it was conjectured [4] that all hereditary classes containing at most $2^{O(n \log n)}$ many n-vertex graphs (*factorial* classes) have an implicit representation (in this case of size $O(\log n)$). Hatami & Hatami [3] recently disproved this conjecture by using the probabilistic method to construct a hereditary factorial class requiring almost $\sqrt{n}$ size labels. We show that this conjecture also fails for *monotone* (closed under subgraphs) factorial classes, by constructing such classes which require $\Omega(\log^2 n)$ length labels, and we prove a matching upper bound [1].

We conjecture that any hereditary *small* (containing at most $n!\, c^n$ graphs on n vertices, for some constant $c > 0$) class admits an implicit representation with labels of size $O(\log n)$. We show [2] that some constant (depending on c) is needed in front of the $\log n$. Additionally, we prove two restricted cases of this conjecture, namely that monotone small classes and *weakly-sparse* ($K_{t,t}$-free for some constant t) hereditary small classes have implicit representations. In fact, we show a more general result: any monotone or weakly-sparse small class of graphs has bounded degeneracy.

This is joint work with Édouard Bonnet, Julien Duron, Viktor Zamaraev and Maksim Zhukovskii.

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Konstanty Junosza-Szaniawski
Warsaw University of Technology

Partial packing coloring of a triangular-square grid

The concept of packing coloring in graph theory is motivated by the frequency assignment problem in radio networks. This approach entails assigning positive integers to vertices, with the requirement that for any given label (color) i, the distance between any two vertices sharing this label must exceed i. After over 20 years of intensive research, the minimal number of colors needed for packing coloring of an infinite square grid has been established to be 15 [1]. Moreover, it is known that a hexagonal grid requires a minimum of 7 colors for packing coloring, and a triangular grid is not colorable with any finite number of colors in a packing way [2]. Actually no more than fraction 82,2%, (but at least 72,,8%) of the triangular grid can be colored in a packing way with a finite number of colors.

In this talk we consider packing colorings of triangular-square grids (there are two such grids). We preset for both grids upper and lower bounds on the fraction of the grid that can be colored in a packing way with a finite number of colors.

This is joint work with Hubert Grochowski and Zuzanna Maciejewska.

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William T. Trotter

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The class of posets with planar cover graphs is dim-bounded

Resolving a conjecture that dates from the 1970's, we show that dimension is bounded in terms of standard example number for the class of posets that have a planar cover graph. At the 2022 PCC, we reported that we could settle this conjecture in the affirmative for the special case of posets with a unique minimal element. While the argument for the general result builds on this case, the proof is complex, requiring an additional 60 pages, and it has taken us two years to write it all down.

This is joint work with Heather Smith Blake, Jędrzej Hodor, Piotr Micek and Michał Seweryn.

Bartosz Walczak

Jagiellonian University

Coloring graphs with an excluded Burling graph

A class of graphs $\mathcal{G}$ is χ-bounded if there is a function $f\colon \mathbb{N} \to \mathbb{N}$ such that every graph in $\mathcal{G}$ with chromatic number greater than $f(k)$ contains a k-clique. Several important classes of graphs, including the class of string graphs and, more generally, every class of graphs excluding induced subdivisions of a fixed graph, are not χ-bounded because they contain a specific infinite family $\{B_k\}_{k=1}^{\infty}$ of triangle-free graphs with unbounded chromatic number, first constructed by Burling [1]. We show that if $\mathcal{G}$ is the class of string graphs or any 2-*controlled* class of graphs excluding induced subdivisions of a fixed graph, Burling's construction is the only reason for $\mathcal{G}$ not being χ-bounded; that is, there is a function $f\colon \mathbb{N} \times \mathbb{N} \to \mathbb{N}$ such that every graph in $\mathcal{G}$ with chromatic number greater than $f(k, \ell)$ contains a k-clique or an induced copy of B_ℓ. This (up to the "2-controlled" condition) confirms a "wild conjecture" of Chudnovsky, Scott, and Seymour [2, Conjecture 2.1]. Being 2-controlled means that the chromatic number is bounded in terms of the maximum chromatic number of a ball of radius 2; this holds for string graphs and is conjectured to hold for every class of graphs excluding induced subdivisions of a fixed graph.

This is joint work with Tara Abrishami, Marcin Briański, James Davies, Xiying Du, Jana Masaříková, and Paweł Rzążewski.

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Mehmet Akif Yıldız
University of Amsterdam

Path decompositions of oriented graphs

We consider the problem of decomposing the edges of a digraph into as few paths as possible. It is not too hard to see that a natural lower bound for the number of paths in any path decomposition is $\frac{1}{2}\sum_{v\in V(D)}|d^+(v)-d^-(v)|$. Any digraph that achieves this bound is called consistent. As a generalization of Kelly's conjecture on Hamilton decompositions of regular tournaments, Alspach, Mason, and Pullman [1] conjectured in 1976 that every tournament of even order is consistent. This was recently verified for large tournaments by Girão, Granet, Kühn, Lo, and Osthus [2]. A more general conjecture of Pullman [3] states that every orientation of a regular graph of odd degree is consistent. We prove that such a statement holds for a random regular graph of odd degree with high probability. Along the way, we verify the conjecture for graphs with no short cycles.

This is joint work with Viresh Patel.

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Andrzej Żak
AGH University

Coloring the input of a knapsack problem

We investigate the impact of input coloring on the knapsack problem and demonstrate that a combinatorial-like approach can effectively overcome significant computational complexity challenges. Our proposed solution has been successfully implemented in real-world order acceptance and scheduling scenarios.

This is joint work with Marcin Anholcer.

Rui-Ray Zhang
Barcelona School of Economics

Residual entropy and Eulerian orientations of graphs and random graphs with given degrees

By investigating Pauling's mean-field approximation, we study the Eulerian orientations of certain sparse and dense random graphs with given degrees. This corresponds to the residual entropy of ice-type models on those graphs. For a wide range of regular graphs, we observe a negative correlation between the residual entropy and spanning tree entropy.

This is based on joint works with Mikhail Isaev and Brendan McKay.

Huan Zhou

Zhejiang Normal University, Paris Cité University

The Alon-Tarsi number of planar graphs with girth 5 and locally planar graphs

In this talk, we study the Alon-Tarsi number of planar graphs with girth 5 and locally planar graphs. We proved that the Alon-Tarsi number of any planar graph with girth 5 is at most 3, and that of any locally planar graph is at most 5. These results were proved by using a stronger graph coloring parameter, weak degeneracy, which is motivated by a modified greedy coloring algorithm.

This is joint work with Ming Han, Tao Wang, Jianglin Wu and Xuding Zhu.

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Jialu Zhu
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Minimum non-chromatic-choosable graphs

A graph G is called chromatic-choosable if $\chi(G) = \mathrm{ch}(G)$. A natural problem is to determine the minimum number of vertices in a k-chromatic non-k-choosable graph. It was conjectured by Ohba, and proved by Noel, Reed and Wu that k-chromatic graphs G with $|V(G)| \leq 2k+1$ are k-choosable. This upper bound on $|V(G)|$ is tight. It is known that if k is even, then $G = K_{3\star(k/2+1),1\star(k/2-1)}$ and $G = K_{4,2\star(k-1)}$ are k-chromatic graphs with $|V(G)| = 2k+2$ but they are not k-choosable. Firstly, we can prove that all other k-chromatic graphs G with $|V(G)| = 2k+2$ are k-choosable. In particular, if $\chi(G)$ is odd and $|V(G)| \leq 2\chi(G)+2$, then G is chromatic-choosable, which was conjectured by Noel.

For a multi-set $\lambda = \{k_1, k_2, \ldots, k_q\}$ of positive integers, let $k_\lambda = \sum_{i=1}^{q} k_i$. A λ-list assignment of G is a list assignment L of G such that the colour set $\bigcup_{v \in V(G)} L(v)$ can be partitioned into the disjoint union $C_1 \cup C_2 \cup \ldots \cup C_q$ of q sets so that for each i and each vertex v of G, $|L(v) \cap C_i| \geq k_i$. We say G is λ-choosable if G is L-colourable for any λ-list assignment L of G. The concept of λ-choosability puts k-colourability and k-choosability in the same framework: If $\lambda = \{k\}$, then λ-choosability is equivalent to k-choosability; if λ consists of k copies of 1, then λ-choosability is equivalent to k-colourability. If G is λ-choosable, then G is k_λ-colourable. On the other hand, there are k_λ-colourable graphs that are not λ-choosable, provided that λ contains an integer larger than 1. Let $\phi(\lambda)$ be the minimum number of vertices in a k_λ-colourable non-λ-choosable graph. Finally, we can determines the value of $\phi(\lambda)$ for all λ.

This is joint work with Xuding Zhu.

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Indicated list colouring of graphs

Assume G is a graph and L is a list assignment that assigns to each vertex v of G a set $L(v)$ of permissible colours. The *indicated L-colouring game* on G is played by two players: Ann and Ben. In each round, Ann chooses an uncoloured vertex v and Ben assigns to v a colour $\phi(v) \in L(v)$ that has not been assigned to any of its neighbors. Ann wins the game if all vertices of G are coloured. Otherwise, at a certain round, there is an uncoloured vertex v such that all colours in $L(v)$ are assigned to neighbours of v, and Ben wins the game. We say G is *indicated L-colourable* if Ann has a winning strategy for the indicated L-colouring game. A *degree-list assignment* of G is a list assignment L with $|L(v)| \geq d_G(v)$ for each vertex v. We say G is indicated degree-choosable if G is indicated L-choosable for any degree-list assignment L of G. This paper characterizes all the pairs (G, L) such that L is a degree-list assignment of G and G is not indicated L-colourable. Then we prove a Brooks Theorem for indicated chromatic number: characterize all graphs G with indicated chromatic number greater than its maximum degree.

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